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ACOUSTIC RESONANCE IN SUBSONIC AERODYNAMIC INTERACTION OF CASCADES

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It is known [1] that acoustic resonance could take place in turbomachinery cascades when frequencies of any periodic disturbances coincide with characteristic frequencies of flow fluctuations in cascades. The results of studies on this phenomenon are given in [2, 3] for the case when acoustic disturbances are caused by fluctuations in flow in the trailing-edge wakes. However, the most powerful, constantly acting, and periodic source of disturbances in turbomachines is the aerodynamic interaction of the impeller and the guide vanes. The present work is devoted to the experimental and theoretical determination of conditions for its appearance.

1. Consider two annular cascades with one of them rotating about the axis of symmetry z at an angular velocity Ω. Introduce a stationary cylindrical coordinate system $(\cdot, \theta, z)$ and also a moving system (r, θ_1, z) rigidly fixed to the rotating cascade so that

$$\theta = \theta_1 + \Omega t. \tag{1.1}$$

When the flow past each of these cascades is uniform, the velocities are periodic functions of θ with periods $2\pi/N$, $2D/N_1$, where N and N_1 are the number of blades in the stator and rotor cascades, respectively, i.e.,

$$V(r, \theta, z) = \sum_{n=-\infty}^{\infty} v_n(r, z) \exp(inN\theta),$$

$$V_1(r, \theta_1, z) = \sum_{n=-\infty}^{\infty} v_{1n}(r, z) \exp(inN_1\theta_1).$$

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In the stationary coordinate system the velocity field V_1 has the following form in view of Eq. (1.1)

$$V_1 = \sum_{n=-\infty}^{\infty} v_{1n} \exp [inN_1 (\theta - \Omega t)], \quad (1.2)$$

which is a system of traveling waves along the circumference.

The stator cascade located in this field will obviously experience periodic effect which, in the linear approximation, reduces to acoustic disturbances of the fluid with a frequency spectrum $\lambda_{1n} = nN_1\Omega$ ($n = \pm 1, \pm 2, \ldots$). In their turn, the rotor blades, interacting with the velocity field V, represent a system of acoustic disturbance sources with the frequency spectrum $\lambda_n = nN\Omega$ ($n = \pm 1, \pm 2, \ldots$).

If it is assumed that the mean steady flow is potential and isentropic, then, according to [4], the determination of the velocity potential of the above-mentioned acoustic disturbances could be reduced to the solution of the nonhomogeneous wave equation with variable coefficients

$$\frac{\partial^2 \Phi}{\partial t^2} - L(\Phi) = F_1(t, \rho) + F_2(t, \rho) \quad (1.3)$$

with homogeneous Neumann conditions at the blade surfaces and in the presence of radiation. Here F_1 and F_2 are functions that describe physical no-slip conditions on the blades with aerodynamic interaction of cascades.

In what follows we shall consider only those components of acoustic disturbances which arise as a result of interaction of the stator blades with the velocity field (1.2) created by the rotor blades. The corresponding function $F_1(t, \rho)$, taking into consideration the variable coefficients of Eq. (1.3) due to the velocity field V, is expressed in a general form as follows:

$$F_1 = \sum_{n=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f_{ns}(r, z) \exp \{i [(nN_1 + sN)\theta - \lambda_{1n} t]\},$$
$$\lambda_{1n} = nN_1\Omega. \quad (1.4)$$

Applying the principle of superposition to the solution of Eq. (1.3), an elliptic equation in the respective amplitude function φ_n may be obtained for each harmonic on the right-hand side of Eq. (1.4)

$$\widetilde{L}_n(\varphi_n) + \lambda_{1n}^2 \varphi_n = \widetilde{F}_{1n} \quad (n = 0, \pm 1, \pm 2, \ldots), \quad (1.5)$$

where

$$\widetilde{F}_{1n} = e^{inN_1\theta} \sum_{s=-\infty}^{\infty} f_{ns}(r, z) e^{isN\theta}. \quad (1.6)$$

Expand $\widetilde{F}_{1n}$ in terms of eigenfunctions for the flow fluctuations across the given blade assuming a complete set of these functions:

$$\widetilde{F}_{1n} = \sum_{p=1}^{\infty} \sum_{m=1}^{N-1} c_{pm}^{(n)} \psi_{pm}. \quad (1.7)$$

It is known [1] that eigenfunctions for the above problem have a generalized periodicity along the arc, i.e., have a form

$$\psi_{pm} = \sum_{l=-\infty}^{\infty} \sigma_{pml}(r, z) \exp [i(lN + m)\theta]. \quad (1.8)$$

In general, the respective eigenvalues of the problem k_{pm} are complex and their set is discrete [5].

Assuming $\|\psi_{pm}\| = 1$, we find from (1.7)

$$c_{pm}^{(n)} = \int_D \tilde{F}_{1n}\bar{\psi}_{pm}dv,$$

where D is the region of the solution to the problem; $\bar{\psi}_{pm}$ is the complex conjugate function of ψ_{pm}. If the integral along the region D is reduced to a periodic integral, then the interaction with respect to the variable θ using Eqs. (1.6) and (1.8) leads to

$$\int_0^{2\pi} \tilde{F}_{1n}\bar{\psi}_{pm}d\theta = \begin{cases} 0 & \text{for} \quad nN_1 + jN \neq m, \\ 2\pi f_{ns}\sigma_{pmi} & \text{for} \quad nN_1 + jN = m \end{cases} \tag{1.9}$$

($j = 0, \pm 1, \pm 2, \ldots$).

Expressing the solution to Eq. (1.5) also in the form of a series of eigenfunctions, and using Eq. (1.7) we get

$$\varphi_n = \sum_{p=1}^{\infty}\sum_{m=1}^{N-1} \frac{c_{pm}^{(n)}\psi_{pm}}{\lambda_{1n}^2 - (k_{pm}a/b)^2}.$$

Here a is the velocity of sound in the freestream and b is the reference length of blades.

In view of Eq. (1.9), the terms that are equal to zero in this expression are those which do not satisfy the condition

$$0 < m = nN_1 + jN < N \quad (n, j = 0, \pm 1, \ldots). \tag{1.10}$$

It follows that acoustic resonance in the flow through the stator blades due to the interaction of blades could occur only when

$$\lambda_{1n} = nN_1\Omega = \omega_{pm}^*,$$

where $\omega_{pm}^* = \mathrm{Re}\,(k_{pm}^* a/b)$ is the eigenfrequency of fluid fluctuations and the corresponding eigenfunction satisfies (1.10).

It is more convenient to express (1.10) in the form

$$0 < \mu_m = 2\pi\left[n\frac{N_1}{N} + j\right] < 2\pi \quad (n, j = 0, \pm 1, \ldots) \tag{1.11}$$

(μ_m is the phase-shift parameter of the characteristic flow fluctuations through the cascade).

2. The phenomenon of acoustic resonance caused by the interaction of annular cascades was first discovered experimentally in the Kalinin Leningrad Physics Institute (LPI) compressor design laboratory while investigating unsteady processes in centrifugal compressor stages [6]. More careful studies of this phenomenon were conducted using special apparatus [7] with a miniature integrating semiconductor pressure transducer having a characteristic frequency range of 50-70 kHz, a differential broadband constant current amplifer, and sensors to determine compressor rotor frequency.

The signals were recorded in a multiple beam oscilloscope S1-33, rotation frequency was measured by the frequency meter Ch3-33, and the measurement cycle was synchronized with flash from the photographic camera RFK.

The apparatus ensures the measurement of rapidly varying fluctuations in static pressure in the frequency range 0-10 kHz with dynamic error not exceeding 5%. The conventional measurement techniques [8] were used to determine the quantities required to obtain the aerothermodynamic characteristics of the compressor.

The object of study was a single-stage centrifugal compressor whose rotor diameter D_0 = 0.275 M with number of blades N_1 = 16 and exit angle β = 49°. The number blades in the diffuser was varied (N = 24, 19, 13, and 7). The inlet and exit diameters of the diffuser D_1 = $1.09D_0$ and D_2 = $1.43D_0$ remained unchanged, as were the geometric parameters of the blade sections (the mean line of blade sections was a circular arc with angles at inlet and exit being 20 and 32, respectively).

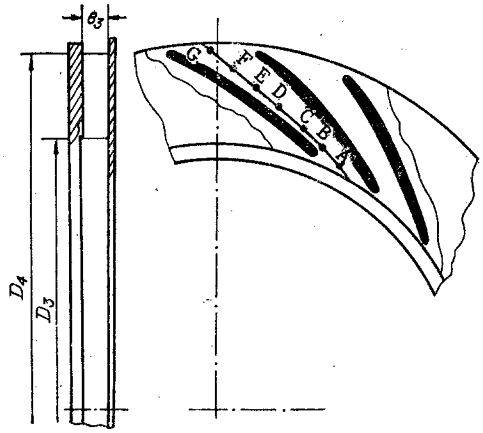

Fig. 1

In the experiment, fluctuating pressures were measured at specific points in the inter-stage ducts of the rotor and diffuser, at fixed angles of attack at the cascade inlet, and for different values of mass flow. The location of static-pressure sensors (A...G) at the front wall of the diffuser along the duct centerline is shown in Fig. 1.

Typical oscillograms of pressure fluctuations at characteristic impeller sections for maximum flow rate are shown in Fig. 2. When the impeller rotational speed is varied gradually, appreciable increase in amplitudes of fluctuation in the duct is observed at certain conditions ($f = N\Omega = 2520$ Hz) and have a right sinusoidal form. The amplitudes of fluctuations are small outside these conditions and the nature of fluctuations is less periodic.

The inlet conditions for the variations in relative amplitudes of pressure fluctuations in the diffuser are shown in Fig. 3 with $N = 24$, $\varphi_2 = 0.28$, 0.22, and 0.16 at angles-of-attack $\alpha = -11$, 0, and 8° (a-c). It is seen that the increase in pressure fluctuations at certain conditions has a clear resonant character. It is worth noting here that at 1640 Hz and 3300 Hz the amplitude of fluctuations increases most significantly at the center of the duct and, at 1900 Hz, only at the end of the duct. A similar situation exists even for blade diffusers with $N = 19$ and 13.

The last condition, and also the fact that the nature of fluctuations is nearly harmonic (Fig. 2) in the presence of increased pressure fluctuations, indirectly suggest that the observed phenomenon is acoustic resonance. However, a sound basis for this interpretation of experimental data is given by results of the comparison of the frequencies of fluctuations at the above conditions with the eigenfrequencies of the fluctuations in blade diffusers.

3. Consider, in the increasing order of complexity, four design models to determine eigenfrequencies of fluid fluctuations in a bladed diffuser which is considered the object of the study since the corresponding problem for rotating cascades is not yet solved.

It is possible to most simply estimate the eigenfrequencies by modeling the blade diffuser as a straight duct whose length ℓ is equal to the mean line of interblade passage. Then the eigenfrequency is determined by the equation

$$f = \frac{ma\left(1 + M_c^2\right)}{2\left(l + \gamma_1 + \gamma_2\right)} \quad (m = 1, 2, \ldots), \tag{3.1}$$

where M_c is the mean flow Mach number in the duct; γ_1, γ_2 are corrections for the open ends at the inlet and outlet. According to [9], $\gamma_j \approx 0.6R_j$ (R_j is the radius of the opening).

The second model has a more complex geometry of the region (Fig. 4a) but the corresponding mathematical problem reduces to the determination of the reduced frequency parameter of the fluid fluctuations when a nontrivial solution to the Helmholtz equation exists in the region outside the plates with homogeneous Neumann conditions on the plates, radiation conditions, and a generalized periodicity condition along the circumferential coordinate.

In the third and fourth models, characteristic fluctuations of the fluid are considered in the neighborhood of the plane circular cascade (Fig. 4b), which is comprised of segments from logarithmic spirals with and without the consideration of basic stationary flow, respectively. In the last model it was assumed that the velocity of the basic stationary flow,

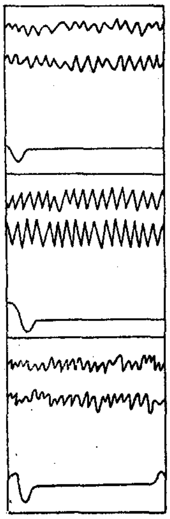

Fig. 2

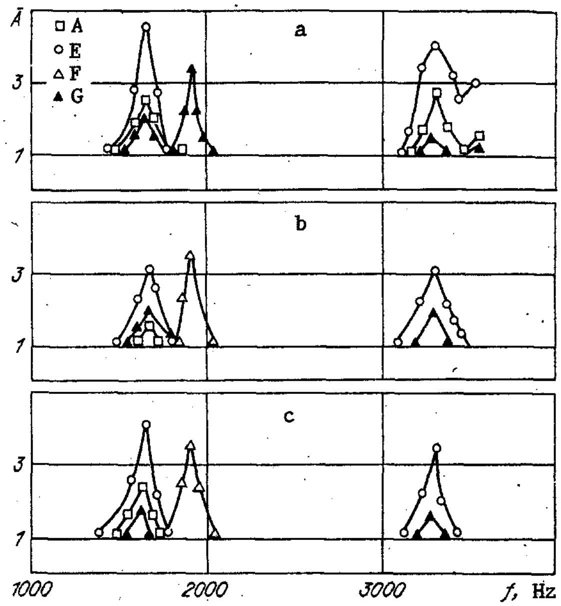

Fig. 3

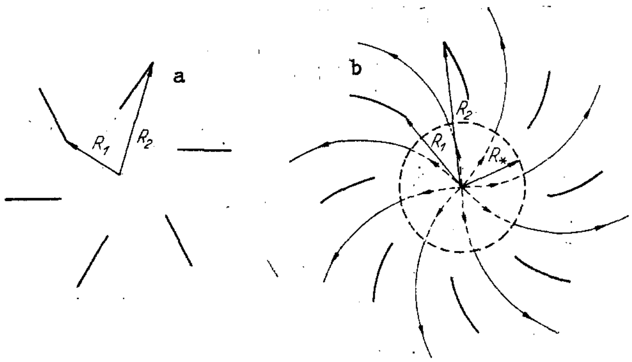

Fig. 4

TABLE 1

N	M	k_e	k_1	k_2	k_3	k_4
24	0,27	4,68	3,99	4,40	$5{,}19 - i\,0{,}12$	$4{,}80 - i\,0{,}25$
	0,31	5,43	7,78	8,99	$7{,}13 - i\,0{,}14$	$5{,}55 - i\,0{,}45$
	0,55	9,42	9,00	13,55	$11{,}52 - i\,0{,}17$	$9{,}26 - i\,0{,}72$
13	0,23	3,99	3,88	4,40	$5{,}10 - i\,0{,}15$	$4{,}12 - i\,0{,}52$
	0,32	5,48	7,35	8,99	$6{,}33 - i\,0{,}17$	$5{,}18 - i\,0{,}63$
	0,37	6,28	10,61	11,48	$8{,}75 - i\,0{,}25$	$6{,}54 - i\,0{,}70$
	0,44	7,62	13,20	13,55	$9{,}91 - i\,0{,}19$	$7{,}73 - i\,0{,}91$
	0,53	9,14	14,73	15,71	$10{,}90 - i\,0{,}31$	$9{,}16 - i\,1{,}00$

which is a spiral flow, in the interblade passages is sufficiently small that when $r > R_*$ ($R_* < R_1$), the square of the local Mach number can be neglected. As far as acoustic disturbances are concerned, it is assumed that their sources are absent when $r \le R_*$ and $r > R_2$.

Within the framework of these assumptions the problem of characteristic (eigen) flow fluctuations of the fluid through circular cascade is reduced [11] to the solution of the equation

$$\Delta u + k^2 u + 2ik\left(\frac{\sigma}{r}\frac{\partial u}{\partial r} + \frac{\delta}{r^2}\frac{\partial u}{\partial \theta}\right) = 0. \tag{3.2}$$

Here u is the amplitude function for the nonstationary flow component; $\sigma = Q/2\pi a R_1$, $\delta = \Gamma/2\pi a R_1$; Q and Γ, source strength and circulation of the vortex that induce the basic stationary flow, $\bar{r} = r/R_1$, $k = \omega R_1/a$; ω, frequency of fluctuations of the fluid. The parameter sought is k, that corresponds to a nontrivial solution of Eq. (3.2), satisfying the following boundary conditions:

Homogeneous, no-slip boundary conditions on the blade profiles

$$\frac{\partial u}{\partial \nu_j} = 0, \quad (r, \theta) \in L_j \quad (j = 1, 2, \ldots, N-1) \tag{3.3}$$

(ν_j is the normal to L_j, L_j is the contour of the j-th blade);

generalized periodicity condition

$$u(r, \theta + \alpha) = e^{i\mu_m} u(r, \theta)$$
$$(\alpha = 2\pi/N, \quad \mu_m = m\alpha \quad (m = 1, 2, \ldots, N-1)); \tag{3.4}$$

absence of acoustic disturbances when $r > R_2$

$$u(r, \theta) = e^{-ik\sigma \ln \bar{r}} \sum_{s=-\infty}^{\infty} a_s H_\zeta^{(1)}(k\bar{r}) e^{is\theta}; \tag{3.5}$$

when $R_* < r < R_1$

$$u(r, \theta) = e^{-ik\sigma \ln \bar{r}} \sum_{s=-\infty}^{\infty} b_s J_\zeta(k\bar{r}) e^{is\theta}, \tag{3.6}$$

where $J_\zeta(x)$ is the Bessel function; $H_\zeta^{(1)}(x)$ is the Hankel function of the first type with complex indices $\zeta = \sqrt{(s + k\delta)^2 - k^2(\sigma^2 + \delta^2)}$ [the square root is determined from the condition $\mathrm{Re}(\zeta) \geq 0$].

The solution to this problem is sought in the class of functions that are unbounded at the sharp trailing edges of blade profile and that satisfy the condition of zero-circulation of the unsteady component of the velocity around each profile. When the last condition is satisfied, there will be no vortex wake downstream of the profiles in the cascade. However, as shown by the investigation carried out earlier for straight cascades [12] at low Mach numbers, the inclusion of vortex wakes practically does not affect the values of eigenfrequencies of flow fluctuations.

A comparison of experimental data with the computation for two-bladed diffusers is given in the table where M is the Mach number of the basic steady flow at the inlet to the cascade ($r = R_1$) for which acoustic resonance was experimentally observed at the reduced frequencies $k_e = N_1 \Omega R_1/a$ (a = 340 m/sec).

A comparison of the values of reduced frequency of characteristic fluctuations $k_1 = 2\pi f R_1/a$, computed for the first model from Eq. (3.1) with k_e shows that there is a good agreement at certain conditions of acoustic resonance. However, in the majority of cases, the values of k_1 and k_e significantly differ from each other since the numerical model reflects neither the influence of other blades of the cascade nor the influence of diffusion in the interblade passages on eigenfrequencies. Theoretical estimate of the limit of the applicability of this model has not yet been obtained.

Values of $k_2 = \omega R_1/a$, where ω is the circular frequency of characteristic fluctuations are computed from the asymptotic relation (as $N \to \infty$) for the second model [10]. In this expression the phase-shift parameter that represents the cascade effect is absent. To some extent such a model reflects diffusivity of the interblade passage, but with respect to the other parameters it is even worse than the former.

Complex eigenvalues of the problem (3.2)-(3.6), k_3 are computed for M = 0, and k_4 for values of M corresponding to experimental conditions. Here their real parts determine eigenfrequencies of flow fluctuations and the imaginary parts represent the loss of acoustic energy through radiation to the surrounding space.

It is worth noting that the condition for generalized periodicity (3.4) in computing the above-mentioned quantities was carried out with the inclusion of (1.11), i.e., the phase-shift parameter for the first diffuser (N = 24) is taken equal to $\mu_m = 2\pi N_1/N = 4\pi/3$ (N_1 = 16), and for the second (N = 13), $\mu_m = 0.462\pi$.

The very good agreement of $\mathrm{Re}(k_4)$ and k_e, in our view, indicates that all the principal factors that determine characteristic fluctuations of the fluid in practical designs are included in the theoretical model. Actually, for the first modes of characteristic fluctuations of the fluid in the given diffusers, three-dimensionality is of secondary importance since their dimension in the direction of the axis of rotation is many times less than the length of the interblade passages. The approximate formulation of conditions at the inlet and exit from the cascade (3.5), (3.6) also negligibly affects results, perhaps, because, as shown by the investigation for straight cascade [12], characteristic fluctuations are localized quite close to its neighborhood.

Thus the results of these comparisons lead to the following conclusions.

1. Since conditions for increased pressure fluctuations coincide with conditions at which the fluid flow through the cascade could accomplish characteristic fluctuations, the corresponding phenomena can be interpreted as acoustic resonance.

2. In order to determine these conditions using the available computational techniques with sufficient reliability, it is possible to recommend only the algorithm [11], which is based on the solution to the problem (3.2)-(3.6). Here, if the source of disturbance is the aerodynamic interaction of cascades, the solution to the above-mentioned problem must be sought with additional condition (1.11).

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